gain-bandwidth product curve is less than the calculated values, but there was no way to adjust and optimize the coupling once the loop was formed. In any tunable maser, some method of adjusting the coupling could be easily incorporated to offset this trouble. Fig. 3 shows pump power necessary for saturation vs signal input power. The amount of pump power necessary for saturation is larger than that usually required for the resonant cavity case, but not significantly so. We also noted that the gain of the maser is not dependent upon the frequency stability of the pump source. It was also noted that oscillations could be started over nearly the entire linewidth of the ruby, *i.e.*, 50-Mc tuning range of the pump frequency with little variation in the pump power.

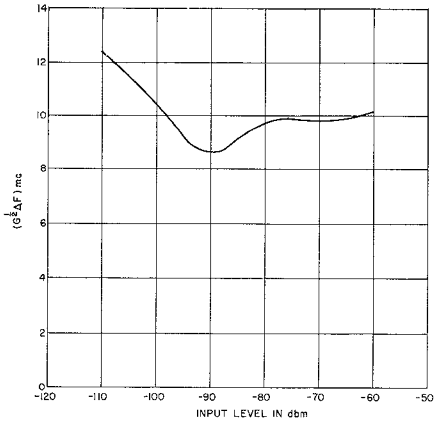

Fig. 2—Gain bandwidth product vs input level in dbm.

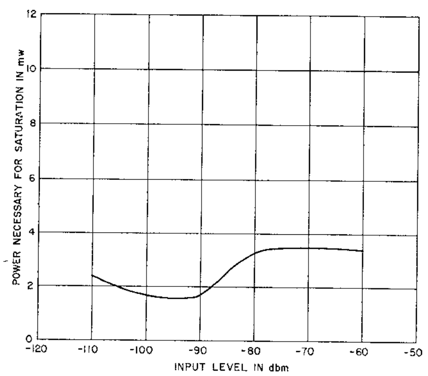

Fig. 3—Saturation pump power vs signal input level in dbm.

In conclusion, it appears feasible that a tunable cavity maser at S band could be developed without recourse to a dual-mode cavity system. This would greatly enhance the practicability of this type of amplifier for the 2000 Mc region where masers can be used efficiently for telemetry purposes.

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Higher-Order Evaluation of Dipole Moments of a Small Circular Disk for Arbitrary Incident Fields*

In a recent note[1] the induced electric and magnetic dipole moments P and M due to the diffraction of a plane wave on a small circular disk were given. The expression for the electric dipole moment holds, however, only if the electric field vector is parallel to the plane of incidence. In a more general approach the case for an arbitrary primary field has been examined, and the following expressions have been obtained:

$$P_x = \frac{16}{3} a^3\epsilon_0 \left[E_x^i + \frac{(ka)^2}{30}\left(13E_x^i - \frac{3}{k^2}\frac{\partial^2 E_x^i}{\partial z^2} + \frac{2j}{\omega\epsilon_0}\frac{\partial H_z^i}{\partial y}\right) - j\frac{8}{9\pi}(ka)^3 E_x^i + 0((ka)^4)\right],$$

$$P_y = \frac{16}{3} a^3\epsilon_0 \left[E_y^i + \frac{(ka)^2}{30}\left(13E_y^i - \frac{3}{k^2}\frac{\partial^2 E_y^i}{\partial z^2} - \frac{2j}{\omega\epsilon_0}\frac{\partial H_z^i}{\partial x}\right) - j\frac{8}{9\pi}(ka)^3 E_y^i + 0((ka)^4)\right],$$

$$M_z = -\frac{8}{3} a^3 \left[H_z^i - \frac{(ka)^2}{10}\left(3H_z^i + \frac{1}{k^2}\frac{\partial^2 H_z}{\partial z^2}\right) + j\frac{4}{9\pi}(ka)^3 H_z^i + 0((ka)^4)\right].$$

The axis of the disk is along the z direction. The incident fields are evaluated at the center of the disk.

We now consider a plane wave ($\boldsymbol{E}^i$, $\boldsymbol{H}^i$) incident in the xz plane and with an angle of incidence θ. Using the expressions above we obtain

$$P_x = \frac{16}{3} a^3 \left[1 + \left(\frac{8}{15} - \frac{1}{10}\sin^2\theta\right)(ka)^2 - j\frac{8}{9\pi}(ka)^3 + \cdots \right] E_x^i,$$

$$P_y = \frac{16}{3} a^3 \left[1 + \left(\frac{8}{15} - \frac{1}{6}\sin^2\theta\right)(ka)^2 - j\frac{8}{9\pi}(ka)^3 + \cdots \right] E_y^i,$$

$$M_z = -\frac{8}{3} a^3 \left[1 - \frac{1}{10}(2 + \sin^2\theta)(ka)^2 + j\frac{4}{9\pi}(ka)^3 + \cdots \right] H_z^i.$$

P_x and M_z agree with the values given in reference [1].

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[1] W. H. Eggimann, "Higher order evaluation of dipole moments of a small circular disk," IRE TRANS. ON MICROWAVE THEORY AND TECHNIQUES, vol. 8, p. 573; September, 1960.

Capacitance Definitions for Parametric Operation*

The purpose of this note is to reconcile the various definitions of nonlinear or equivalent time-varying capacitance which appear in the literature concerned with parametric devices. The problem considered is one of a nonlinear reactive element, let us say a capacitance, which is pumped by strong pump source at frequency f_p and which couples two circuit modes at frequencies f_s and f_i, usually called the signal and idling frequencies. For parametric operation we demand either $f_p=f_i+f_s$ or $f_p=f_i-f_s$. The nonlinear element (*i.e.*, a capacitance) has a charge-voltage characteristic given by

$$q = f(V). \tag{1}$$

Let us imagine that now we have applied a strong pump voltage V_p together with a dc bias voltage V_0 and subsequently we shall be concerned with the behavior upon application of small signal voltage δv at signal and idling frequencies. We can now define several capacitances:

1) The total capacitance C_T is defined as the ratio of total charge to total voltage, or

$$q = C_T V. \tag{2}$$

Obviously from (1)

$$C_T = \frac{f(V)}{V}; \tag{3}$$

this is the definition of capacitance used by Heffner and Wade.[1]

2) The incremental capacitance C_i is defined by

$$\delta q = C_i \delta V. \tag{4}$$

This is the definition used by Rowe.[2]

Two questions arise: first, in definition 1, what is the relationship between the time-varying capacitance produced by the pump alone to that which is effective in producing parametric action; second, what is the relationship between the two definitions of capacitance?

The first question already has been answered,[1] but it is perhaps worthwhile to add a few more details. If the biased capacitance is acted on by the pump voltage alone, then we would measure a charge,

$$q = (C_{T0} + C_{TP})(V_0 + V_p), \tag{5}$$

where we consider the capacitance to have a dc part C_{T0} and a time varying part C_{TP} varying sinusoidally at the pump frequency (for simplicity we will neglect harmonic terms). The time varying part could be measured or could be inferred from a static plot of the nonlinear characteristic. We necessarily demand that the amplitude of the time-varying part be less than or at most equal to the dc part in order to have the

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[1] H. Heffner and G. Wade, *J. Appl. Phys.*, vol. 29, pp. 1321–1331; September, 1958.

[2] H. E. Rowe, "Some general properties of nonlinear elements—II. Small signal theory," PROC. IRE, vol. 46, pp. 850–860; May, 1958.

total capacitance positive. With the neglect of higher harmonic terms, this implies

$$\frac{|C_{TP}|}{C_{T0}} \leq 1. \tag{6}$$

Suppose now in addition to the bias and pump voltage (V_0+V_P), we apply a small signal voltage δv with components at signal and idling frequencies. What is the increment in charge which flows? From (1) we have

$$\delta q = C_T \delta v + \delta C_T V,$$

and

$$\delta C_T = \left.\frac{dC_T}{dV}\right|_{(V_0+V_P)} \delta v. \tag{7}$$

The first term in the equation for charge represents the ac charge produced by the pumped capacitance. It includes the action of the time-varying capacitance produced by the pump voltage mixing with the small voltages at signal and idling frequencies, giving rise to charge components at idling and signal frequencies. The second term represents the effect of the small signal voltages in varying the nonlinear capacitance and the resulting mixing with the large bias and pump voltages also to produce charge components at signal and idling frequencies. It is not difficult to show that this second nonlinear effect is equal in magnitude to the first.

For simplicity, let us assume a linear relationship between capacity and voltage, that is, a second order nonlinearity in the charge-voltage characteristic.

$$C_T = c_0 + cV. \tag{8}$$

Then in terms of the notation of (4) when only the bias and pump voltages are applied, the dc portion of the total capacity is

$$C_{T0} = c_0 + cV_0, \tag{9}$$

and the time-varying portion is

$$C_{TP} = cV_P.$$

In this notation, the substitution of the capacitance characteristic of (8) into (7) gives

$$\delta C_T = c\delta v, \tag{10}$$

and

$$\begin{aligned} \delta q &= (C_{T0} + C_{TP})\delta v + c\delta v(V_0 + V_P) \\ &= (C_{T0} + cV_0)\delta v + 2C_{TP}\delta v. \end{aligned} \tag{11}$$

Thus, in so far as the small signals are concerned, the nonlinear capacitor appears to have a static capacitance of $(C_{T0}+cV_0)$ and a time-varying portion of $2C_{TP}$, that is, twice the value produced by the pump alone. This is the value of the equivalent time-varying capacitance C_3 used by Heffner and Wade.[1]

$$C_3 = 2|C_{TP}|. \tag{12}$$

From (6) we see

$$\frac{C_3}{C_{T0}} \leq 2, \tag{13}$$

where we have compared the ac capacitance seen by the signal and idling frequencies to the static capacitance observed when only the bias and pump are applied. There is no such limit to the ratio of pumped ac to static capacitance if *both* are measured at signal or idling frequencies.

The second question raised concerns the relationship between the definition of total capacitance C_T and incremental capacitance C_i. In the case of the incremental capacitance, we imagine that we have applied the bias voltage V_0 and the pump voltage V_p and then observe the increment in charge produced by applying a small-signal voltage δv with components at signal and idling frequencies.

$$q = \left.\frac{df(V)}{dV}\right|_{V_0+V_P} \delta v = C_i \delta v. \tag{14}$$

Here the incremental capacitance

$$C_i = \left.\frac{df(V)}{dV}\right|_{(V_0+V_P)} \tag{15}$$

has dc and time-varying components. In Rowe's notation[2]

$$C_i = \sum_{n=-\infty}^{\infty} C_n e^{jn\omega_p t}. \tag{16}$$

One might wonder whether this definition included not only the effect of capacitance change at the pump frequency mixing with signal and idle voltage but also the terms due to variation in capacitance produced by the small signal and idling voltages mixing with the pump voltage, an effect which we have seen is of the same magnitude as the first. It is easy to show that the incremental capacitance approach does indeed include both effects and is identical to the total capacitance approach by returning to (7) which can be written,

$$\delta q = C_T|_{(V_0+V_P)}\delta v + \left[\left.\frac{dC_T}{dV}\right|_{(V_0+V_P)} \delta v\right] V, \tag{17}$$

and (3), which relates C_T to the nonlinear characteristic

$$C_T = \frac{f(V)}{V}. \tag{3}$$

With this substitution for C_T (7) becomes

$$\begin{aligned} \delta q &= \left.\frac{f(V)}{V}\right|_{(V_0+V_P)} \delta v \\ &\quad + \left[\left.\frac{df(V)}{dV}\right|_{(V_0+V_P)} - \left.\frac{f(V)}{V}\right|_{(V_0+V_P)}\right]\delta v \\ &= \left.\frac{df(V)}{V}\right|_{(V_0+V_P)} \delta v = C_i \delta v. \end{aligned} \tag{18}$$

Thus, the two definitions are consistent. If one includes only that portion of the incremental capacitance which varies at the pump frequency C_i, one sees from (16) in Rowe's notation,[2]

$$|C_i| = 2|C_1|; \tag{19}$$

and from (11), (12), and (13)

$$C_3 = |C_i| = 2|C_1| = 2|C_{TP}| \leq 2C_{T0} \tag{20}$$

where, as before, C_{T0} represents the static capacitance observed when only the bias and pump voltages are applied.

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A Tunnel-Diode Amplifying Converter*

Sufficient literature has already been presented to establish the fact that tunnel diodes appear to have a bright future as relatively low noise, low power consumption, amplifying devices.[1-4]

In this note it is desired to report the results of using the tunnel diode as a mixing element rather than a single-frequency amplifying device.

In the usual microwave or VHF receiver, superheterodyne principles are most often utilized. In this system the signal is received by some form of antenna, amplified, and then transmitted to a mixer or transmitted directly to the mixer without amplification. In the mixer, the received signal and the local oscillator signal operate usually on a nonlinear variable-resistance device from which the intermediate frequency signal is then derived.

This mixer is usually quite noisy and quite lossy, the diode-noise figures being on the order of 7.0 db and the diode losses being on the order of 6.0 db.

It is very possible to place a negative-resistance amplifier before the mixer to minimize the effects of the mixer noise figure and loss; but the peculiar current-voltage (I-V) characteristic of the tunnel diode presents a much more desirable solution.[5] The RF amplifier and variable resistance mixer can be entirely eliminated and the tunnel diode used to obtain the necessary high-gain and low-noise figure and to convert from the signal frequency down to the IF frequency.

Fig. 1 is representative of the I-V characteristic of the diode used in the experi-

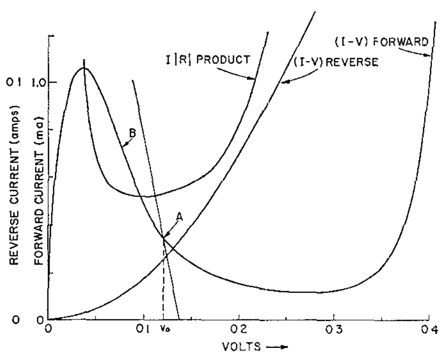

Fig. 1—Current-voltage characteristic of the germanium diode used in the calculations.

* Received by the PGMTT, October 4, 1960. This research was supported by the United States Air Force through WWRNGW of the Wright Air Dev. Div. of the Air Res. and Dev. Command.

[1] H. S. Sommers, Jr., "Tunnel diode as high-frequency devices," Proc. IRE, vol. 47, pp. 1201–1206; July, 1959.

[2] K. K. N. Chang, "Low-noise tunnel diode amplifier," Proc. IRE, vol. 47, p. 1268; July, 1959.

[3] I. A. Lesk, N. Halonyak, U. S. Davidsohn, and M. W. Aarons, "Germanium and silicon tunnel diodes—design, operation and application," 1959 WESCON Convention Record, pt. 3, pp. 9–31.

[4] M. E. Hines, "High-frequency negative resistance circuit principles for Esaki diode applications," *Bell Sys. Tech. J.*, vol. 34, pp. 477–513; May, 1960.

[5] K. K. N. Chang, G. H. Heilmeier, and H. J. Prager, "Low-noise tunnel diode down converter having conversion gain," Proc. IRE, vol. 48, pp. 854–858; May, 1960.